Market Structures :

Monopoly $\longleftrightarrow$ Perf Comp

(1) : $mr = mc$ (many) $p = mc$

Oligopoly : small # of firms (≥ 2)

- Strategic component b/c have $\boxed{\text{market power}}$
- no general sol^n - depends on market structure & # of firms

↳ types : Collusive ⇒ work together
- Cartels (OPEC, diamonds, ...)

Non-Collusive ⇒ individuals

1. Simultaneous
   - Cournot - quantity setting (cement, steel, fert..) choose production cap
   - Bertrand - price setting (gas stations, Ride share)

2. Sequential move
   - Stackberg - leader follower quant setting (ev's, cloud computing...)
   - Price leader - leader follower price setting (telecommunications, streaming ...)

Solving Collusive duopoly (2 firms) cartel

2 firms $i \in \{1, 2\}$ $\underbrace{q_i, q_2}$ each choose q

$P = a - bQ$

$c(q_i) = cq_i \Rightarrow mc_i = c$

$Q = q_1 + q_2$

① Joint monopoly + Cartel

$$\text{Cartel } \pi(q_1+q_2) = P Q - c(q_1) - c(q_2)$$

Joint max π (market level)

Q determines price

$$= (a - bQ)Q - cq_1 - cq_2$$

$$= [a - b(q_1+q_2)](q_1+q_2) - cq_1 - cq_2$$

$$(a + bq_1 + bq_2)(q_1+q_2) - cq_1 - cq_2$$

$$aq_1 + aq_2 + bq_1^2 + bq_1q_2 + bq_2q_1 + bq_2^2 - cq_1 - cq_2$$

FOC: $\frac{\partial \pi}{\partial q_1} = 0 \Rightarrow a - 2bq_1 - 2bq_2 - c = 0$

$$a - c = 2b(q_1+q_2)$$

$$\boxed{\underbrace{q_1 + q_2}_{Q} = \frac{a-c}{2b}}$$

by sym.

$\frac{\partial \pi}{\partial q_2} = 0 \Rightarrow \underbrace{q_1 + q_2}_{Q} = \frac{a-c}{2b}$

Firm 1 + 2 have equal market share

$\boxed{\text{* assume } q_1 = q_2 \text{ *}}$

$$2q = \frac{a-c}{2b}$$

$$q_1 + q_2 = 2q$$

$\hookrightarrow q_1^c = q_2^c = \boxed{\frac{a-c}{4b}}$

$$p^c = a - b\left(\frac{a-c}{2b}\right)$$

market $Q = q_1 + q_2$

$$= a - \frac{a-c}{2}$$

$$\boxed{= \frac{a+c}{2}}$$

→ solution for cartels

if cartel firms follow joint plan

* Suppose firm 1 belives that firm 2 will produce

$$\boxed{q_2 = \frac{a-c}{4b}}$$

↳ now we solve for firms 1 conditional on firm 2:

max indivial Π_1

$\Pi_1(q_1) = Pq_1 - cq_1$

$= [a - b(q_1 + q_2)]q_1 - cq_1$

but this is given I believed.

$\frac{\partial \Pi_1}{\partial q_1} = 0 \Rightarrow a - 2bq_1 - bq_2 - c = 0$

$(a-c) - bq_2 = 2bq_1$

Best q_1 given the value of q_2

reaction / Response function

$$\boxed{q_1 = \frac{a-c}{2b} - \frac{q_2}{2}}$$

* optimal amount for firm 1 given firm 2 *

q_2^c

$$q_1 = \frac{a-c}{2b} - \frac{\left(\frac{a-c}{4b}\right)}{2}$$

$$= \frac{1}{2}\left[\frac{a-c}{b} - \frac{1}{4}\frac{a-c}{b}\right]$$

$$= \frac{1}{2}\left[\frac{3}{4}\frac{a-c}{b}\right]$$

$$= \frac{3}{8}\left(\frac{a-c}{b}\right) \neq \frac{a-c}{4b}$$

$>$

try to max joint π at start

$\therefore$ incentive to deviate ... but so does other & thus cartel not Sustainable structure!

max individual π from begining

Solve non-collusive

(2) Cournot

2 firms - each decide q_i independently $i \in \{1,2\}$

$P = a - bQ$

$C(q_i) = cq_i \Rightarrow MC_i = c$

$Q = q_1 + q_2$

Beliefs important b/c deciding what to produce @ the same time

$$\Pi_1 = Pq_1 - cq_1$$
$$= (a - bQ)q_1 - cq_1$$
$$= [a - b(q_1 + q_2)]q_1 - cq_1$$
$$= (a-c)q_1 - bq_1^2 - bq_1q_2$$

$$\frac{\partial \Pi}{\partial q_1} = 0 \Rightarrow (a-c) - 2bq_1 - bq_2 = 0$$

Best Response/ Reaction function

$$\boxed{q_1^{BR} = \frac{a-c-bq_2}{2b} = \frac{a-c}{2b} - \frac{\tilde{q}_2}{2}}$$

optimal value of firm 1, but depends on q_2 belief!

$$q_2^{BR} = \frac{a-c}{2b} - \frac{\tilde{q}_1}{2}$$ by sym

$\Rightarrow$ Eq^m condition:

(i) firms max Π based on reaction function

(ii) firms beliefs are $\boxed{\text{correct}}$ $q_i = \tilde{q}_i$

$$\therefore q_1 = \frac{a-c}{2b} - \frac{1}{2}\left(\frac{a-c}{2b} - \frac{q_1}{2}\right) = \boxed{q_1^c = \frac{a-c}{3b}}$$

$\tilde{q}_2$

1 eq^n - 1 unkn
↳ Solve

$$\frac{a-c}{(n+1)b} \Rightarrow \frac{a-c}{(2+1)b} = \frac{a-c}{3b}$$

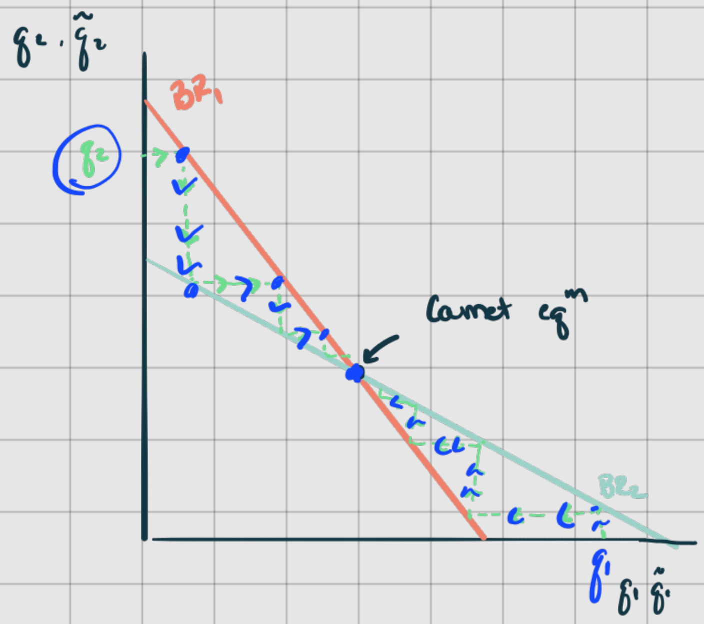
↳ Since cost identical $q_i = q_j$

$$q_1 = \frac{a-c}{2b} - \frac{q_1}{2}$$

$$3q_1 = \frac{a-c}{b}$$

$$q_1^c = \frac{a-c}{3b}$$

Same



Stable eq^m!

↑ move towards cournot eq^m

↳ General Case: (n firms)

$$\begin{bmatrix} P = P(Q) \\ C_i = C(q_i) \end{bmatrix}$$

Firm 1's π max problem:

$Q = q_1 + \tilde{q}_2$

$$\max_{q_1} \pi_1(q_1 \mid \tilde{q}_2) = \underbrace{P(Q)q_1}_{\text{Revenue}} - \underbrace{C_1(q_1)}_{\text{cost}}$$

π_1 given $\tilde{q}_2$

$$\frac{\partial \pi}{\partial q_1} = 0 \Rightarrow \underbrace{P(Q) + \frac{\partial P(Q)}{\partial q_1} q_1}_{MR} - \underbrace{\frac{\partial C_1(q_1)}{\partial q_1}}_{MC_1} = 0$$

$\underline{BR_1} \rightarrow \quad \underbrace{P(Q) + \frac{\partial P}{\partial q_1} q_1}_{MR_1} = MC_1$

MR = MC

$\underline{BR_2} \rightarrow \quad P(Q) + \frac{\partial P}{\partial q_2} q_2 = MC_2$

$\therefore$ to find q_1^c & q_2^c in 2 eq^n system of eq^n

$$\boxed{Q^c = q_1^c + q_2^c \Rightarrow P = P(Q^c)}$$

↳ ≥ 2 firms

$P = P(Q) = a - bQ$

$Q = q_1 + \dots + q_n$

$C_i = c(q_i) \Rightarrow MC_i = c \quad \forall i$

$\rightarrow \quad MR_i = P(Q) + \frac{\partial P}{\partial q_i} q_i = a - bQ - bq_i$

all firms but i

$= a - (bQ_{-i} + bq_i) - bq_i$

$= a - bQ_{-i} - 2bq_i$

like monopoly

$MR_i = MC_i \quad a - bQ_{-i} - 2bq_i = c$

all other firms but i

$q_i = \frac{a-c}{2b} - \frac{Q_{-i}}{2}$ ← Reaction function of firm i

$Q_{-i} \Rightarrow (n-1)q_i$

by Symmetry $(q_i = q_i)$

$$q_i = \frac{a-c}{2b} - \boxed{\frac{(n-1)q_i}{2}}$$

$$(n+1)q_i = \frac{a-c}{b}$$

$$\boxed{q_i = \frac{a-c}{(n+1)b}}$$

$$\therefore Q = nq_i = \frac{a-c}{(n+1)b} \cdot n = \frac{n}{n+1}\,\frac{a-c}{b}$$

$\frac{100000}{100001}$ 100%

$$P = a - b\left(\frac{n}{n+1}\,\frac{a-c}{b}\right) = \frac{a+nc}{n+1}$$

as $n \to \infty$ leads to $P \to c$

* closer to $\boxed{\text{Perfect markets}}$ ← $P = MC$

③ Stackleberg (Leader follower in Q)

(i) firm 1 decides how much to produce (q_1) ←

(ii) firm 2 observes

(iii) firm 2 decide how much to produce (q_2)

↳ ∴ to solve:

how much q_2 given q_1

1) solve firms 2 problem $(q_2(q_1))$

2) incorporate reaction function of firm 2 into firm 1 problem

3) Solve firms 1 problem

Solve F2:

$$P \cdot q_2$$

$$\underline{\max_{q_2}} \ \Pi_2(q_2|q_1) = (a-bQ)q_2 - c(q_2)$$

$$\frac{\partial \Pi_2}{\partial q_2} = 0 \Rightarrow \boxed{q_2^{BR} = \frac{a-c}{2b} - \frac{q_1}{2}}$$

BR2 into $\underline{F1}$:

q_2 is given by BR

$$\max_{q_1} \Pi(q_1, q_2(q_1)) = (a - bQ)q_1 - cq_1$$

$$q_1 + q_2$$

$$= \left[a - b\left(q_1 + \left(\frac{a-c}{2b} - \frac{q_1}{2}\right)\right)\right]q_1 - cq_1$$

$$\frac{\partial \Pi_1}{q_1} = 0 \Rightarrow (a-c) - 2bq_1 - \frac{a-c}{2} + bq_1 = 0$$

$$\frac{a-c}{2} = bq_1$$

$$\boxed{q_1^s = \frac{a-c}{2b}}$$

Solve F2:

BR

$$q_2^s = \frac{a-c}{2b} - \frac{q_1^s}{2} = \frac{a-c}{2b} - \left(\frac{a-c}{2b}\right)\frac{1}{2}$$

$$\boxed{q_s^2 = \frac{1}{4}\frac{a-c}{b}}$$

$$q_1^s + q_2^s = Q^s$$

$$\frac{a-c}{2b} + \frac{a-c}{4b}$$

$$Q^s = \frac{3}{4}\frac{a-c}{b}$$

$$P^s = a + b(Q^s) = a + b\left(\frac{3}{4}\frac{a-c}{b}\right)$$

$$= \frac{a}{4} + \frac{3c}{4}$$

$$\frac{a-c}{2b} > \frac{1}{4}\frac{a-c}{b}$$

Leader has more market power

Leader produces more! First move advantage

④ Bertrand - Simultaneous Price

2 Firms

$C(q_i) = cq_i \Rightarrow MC_i = c$

beliefs

Firm 1 = $\tilde{p}_2 \Rightarrow P_1$ $\Rightarrow$ undercut each other $\Rightarrow P_1 = P_2 = MC$

Firm 2 = $\tilde{p}_1 \Rightarrow P_2$

ex: $P = 100 - Q$

$MC_i = 10$ $AC = 10$

$Q = 100 - P$

F1 $\tilde{p}_2 = 20 \rightarrow$ if

		q_1
$P_1 > P_2$	$q_1 = 0$	$\emptyset$
$P_1 < P_2$	$q_1 =$ all	80
$P_1 = P_2$	$q_1 = q_2$	40 50% 50%

$P_1 = \$19$

$18

$\boxed{MC = P = \$10}$

continually undercut P to reach P = MC

Homework #8 (2,3,4)

Question #2

Duopoly , $MC_1 = 10 + 2Q_1$
$MC_2 = 10 + 2Q_2$
$P = 20 - Q$ $Q = Q_1 + Q_2$

1) Cournot equilibrium : P^c + Q^c

Cournot market structure = firms work independently ∴ they choose output independently + market determines the price @ which its sold

⇒ to solve need to max π for firm 1 + firm 2

$$\max_{Q_i} \pi \quad \pi = P \times Q - \text{Cost}$$

$$\max_{Q_i} \underbrace{(\overbrace{20 - Q_1 - Q_2}^{P})\overset{\text{firm } q}{Q_i}}_{\text{Rev}} - \underbrace{\overset{\text{total cost}}{C_i(Q_i)}}_{\text{total cost}}$$

for firm #1

$$\max_{Q_1} (20 - Q_1 - Q_2)Q_1 - C_1(Q_1)$$

$$\frac{\partial \pi}{\partial Q_1} : 20 - 2Q_1 - Q_2 - \overbrace{10 - 2Q_1}^{\frac{dC(Q)}{dQ} = MC} = 0$$

Q_1 as a function of Q_2

$$4Q_1 = 10 - Q_2$$

$$Q_1(Q_2) = \frac{5}{2} - \frac{1}{4}Q_2 \Rightarrow BR_1$$

Sub into

for firm #2 *By symmetry we know max will be same*

Q_2 as a function of Q_1

$$Q_2(Q_1) = \frac{5}{2} - \frac{1}{4}Q_1$$

↳ taken together can make one equation, one unknown + solve.

Sub $Q_2(Q_1)$ into $Q_1(Q_2)$

$$Q_1^c = \frac{5}{2} - \frac{1}{4}\left[\frac{5}{2} - \frac{1}{4}Q_1\right] = \frac{5}{2} - \frac{5}{8} + \frac{1}{16}Q_1^c$$

$$Q_1^c - \frac{1}{16}Q_1^c = \frac{15}{8}$$

$$\frac{15}{16}Q_1^c = \frac{15}{8}$$

$$\boxed{Q_1^c = 2}$$

⇒ by symmetry Q_2 is same.

$$Q_2^c = 2$$

$$Q = q_1 + q_2 = 4$$

$$P^{\$} = 20 - Q = 20 - 2 - 2 = 16$$

∴ Cournot eq^m is where

firm 1 : $Q_1 = 2$

firm 2 : $Q_2 = 2$

↳ market $Q = 4$ & $P = \$16$

2) Cartel P^{cart} & Q^{cart}

Cartel market structure is where many firms act like one!

aka act like monopoly

↳ ∴ to solve need to include Q & cost for both into one max problem

$\max_Q \pi$ ⇒ Joint π max for all firms.

$$\max_{Q_1, Q_2} \; (20 - Q_1 - Q_2)\underbrace{(Q_1 + Q_2)}_{} - \underbrace{C_1(Q_1) - C_2(Q_2)}_{}$$

$$\frac{\partial \pi}{\partial Q_1} : 20 - 2Q_1 - 2Q_2 - 10 - 2Q_1 = 0 \quad ①$$

$$\frac{\partial \pi}{\partial Q_2} : 20 - 2Q_2 - 2Q_1 - 10 - 2Q_2 = 0 \quad ②$$

sub

① $Q_1 = \frac{5}{2} - \frac{1}{2}Q_2$ ② $Q_2 = \frac{5}{2} - \frac{1}{2}Q_1$

Sub ② into ①

$$Q_1^{cart} = \frac{5}{2} - \frac{1}{2}\left[\frac{5}{2} - \frac{1}{2}Q_1^{cart}\right] = \frac{5}{2} - \frac{5}{4} - \frac{1}{2}Q_1^{cart}$$

$$Q_1^{cart} = \frac{5}{4} + \frac{1}{4}Q_1^{cart}$$
$$\frac{3}{4}Q_1^{cart} = \frac{5}{4}$$
$$\boxed{Q_1^{cart} = \frac{5}{3}}$$

$$Q_2^{cart} = \frac{5}{2} - \frac{1}{2}\left(\frac{5}{3}\right) = \frac{10}{6}$$
$$\boxed{Q_2^{cart} = \frac{5}{3}}$$

$$P^{cart} = 20 - \frac{5}{3} - \frac{5}{3} = \$\underline{16.67}$$

higher P
lower Q

[higher than Cournot]

∴ the Cartel equilibrium occurs when firm 1 $Q_1 = 5/3$, firm 2 $Q_2 = 5/3$
↳ the market $Q = \frac{10}{3}$ & $P = \$16.67$

↖ 3.3

3) Perfect Competition $\underline{Q + P}$.

Solve each independently given Price

$P = MC_1 = 10 - 2Q_1$ $\qquad$ $P = MC_2 = \underline{10 - 2Q_2}$

the individual supply curve: * Same by symmetry

$$S_i(P) = \frac{1}{2}P - 5$$

Market level Supply: $2 \cdot S_i(P) \Rightarrow Q = P - 10$

To solve Q & P: $\boxed{S = D}$

$$P - 10 = 20 - P$$
$$2P = 30$$
$$P^* = 15$$

$$Q = P - 10 = 15 - 10$$
$$Q = 5$$

lowest P
highest Q

In a perfect Competitive market the $P^* = \$15$ & $Q^* = 5$

[lower than cartel & Cournot]

4) Bertrand eqm P & Q

Bertrand of 2 firms = Perfectly Competitive market

$P^B = \$15$, $Q^B = 5$

Question #3

Oligopoly - firms choose q

$P = a - b(q_1 + q_2)$

$mc = c$

1) Cartel Q^{cart} & P^{cart}

Joint π max

$$\max_{q_1 q_2} \left[a - b(q_1 + q_2)\right](q_1 + q_2) - C_1(q_1) - C_2(q_2)$$

$$\left.\begin{aligned} \frac{\partial \pi}{\partial q_1} &: a - 2bq_1 - 2bq_2 = c \\ \frac{\partial \pi}{\partial q_2} &: a - 2bq_2 - 2bq_1 = c \end{aligned}\right] \rightarrow q_1 + q_2 = ?$$

↳ we want to solve for Q^{cart}, which is equal to $(q_1 + q_2)$

⇒ therefore we want to rearrange the derivative to find $q_1 + q_2$

$$a - 2bq_2 - 2bq_1 = c$$

$$a - c = 2bq_2 + 2bq_1$$

$$a - c = 2b(q_2 + q_1)$$

$$\boxed{\frac{a-c}{2b} = q_2 + q_1 = Q^{cart}}$$

Only interested in determining market Q

$$P^{cart} = a - b\left(\frac{a-c}{2b}\right) = \boxed{\frac{a+c}{2}}$$

Cartel eq^m: $Q^{cart} = \frac{a+c}{2b}$ $\quad P^{cart} = \frac{a+c}{2}$

Leader follower

2) Stackleberg eq^m $\underline{Q_1^S, Q_2^S, P^S}$

Stackleberg market structure is a leader follower game, where firm 1 acts first & firm 2 followers

To solve we start w/ firm 2, that is because we don't know what firm 1 will do exactly but we can derive an optimal response function for firm 2 (ie we know how firm 2 will respond)

No BR, b/c firm 1 does not Respond to anything.

Follower

$$\max_{q_2} \ [a-b(q_1+q_2)]q_2 - c(q_2)$$

$$\frac{\partial \pi}{\partial q_2}: a - bq_1 - 2bq_2 - c = 0$$

$$q_2(q_1) = \frac{a-c}{2b} - \frac{q_1}{2}$$

how firm 2 will response, response function for firm 2

Leader

Now that we know how firm 2 will respond, we can calculate what is optimal for firm 1

$$\max_{q_1} \ [a-b(q_1+q_2)]q_1 - c(q_1)$$

$$= aq_1 - \frac{b}{2}q_1^2 - \frac{a-c}{2}q_1 - c(q_1)$$

$$= \left(a - c - \frac{a-c}{2}\right)q_1 - \frac{b}{2}q_1^2$$

$$= \left(\frac{a-c}{2}\right)q_1 - \frac{b}{2}q_1^2$$

Simplifying

$$\frac{\partial \pi}{\partial q_1}: \frac{a-c}{2} - bq_1 = 0 \quad \rightarrow \text{Plug into } q_2(q_1)$$

$$\boxed{q_1^s = \frac{a-c}{2b}}$$

$$q_2^s = \frac{a-c}{2b} - \frac{1}{2}\left(\frac{a-c}{2b}\right)$$

$$\boxed{q_2^s = \frac{a-c}{4b}}$$

$$P^S = a - b\left(\frac{a-c}{2b} + \frac{a-c}{4b}\right) = \frac{a - 3(a-c)}{4}$$

$$\boxed{P^S = \frac{a+3c}{4}}$$

The Stackleberg eq^m is when the leader produces $Q_1 = \frac{a-c}{2b}$ & the follower produces $Q_2 = \frac{a-c}{4b}$. Resulting in $P^S = \$\frac{a+3c}{4}$

3) $\frac{\pi_1}{\pi_2}$ ← ratio of π

per unit π

$\pi = R - C$

$= (\text{per unit } \pi)\, Q$

$$\frac{\pi_1}{\pi_2} = \frac{(P^S - \cancel{c})q_1}{(P^S - \cancel{c})q_2} = \frac{\frac{a-c}{2b}}{\frac{a-c}{4b}} = 2$$

The leader earns twice as much!

Question # 4

$P = 50 - 5Q \qquad MC = 0 \ \forall i$

Stackleberg $Q_1 = ?$

Follower

$$\max_{q_2} (50 - 5q_1 - 5q_2)q_2 - \cancel{c(q_2)} \quad MC=0$$

$$\frac{\partial \pi}{\partial q_2}: 50 - 5q_1 - 10q_2 = 0$$

$$\boxed{q_2(q_1) = 5 - \frac{1}{2} q_1}$$ → Plug in Firm 1 problem

leader

$$\max_{q_1} \left[50 - 5q_1 - 5\left(5 - \frac{1}{2}q_1\right)\right] q_1 - \cancel{c}(q_1)$$

mc=0

$$\frac{\partial \pi}{\partial q_1} : 50 - 10q_1 - 25 + 5q_1 = 0$$

$$\boxed{q_1^s = 5}$$

$$q_2 = 5 - \frac{1}{2}5 \Rightarrow 2.5$$

$$p^s = 50 - 5(7.5)$$

Market Structures

Monopoly — 1 firm — highest Price

Cartel

Cournot

Stackelberg

Bertrand

Competition — many firms — highest Quantity