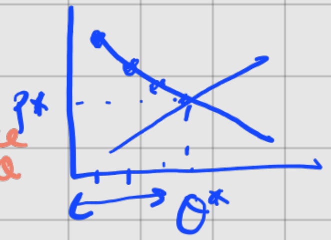
MONOPOLY :

Competive Market ⟷ Monopoly

- price takers
- many firms
- linear Rev ← same price for all Q
- $P = mc$ $(\pi = 0)$



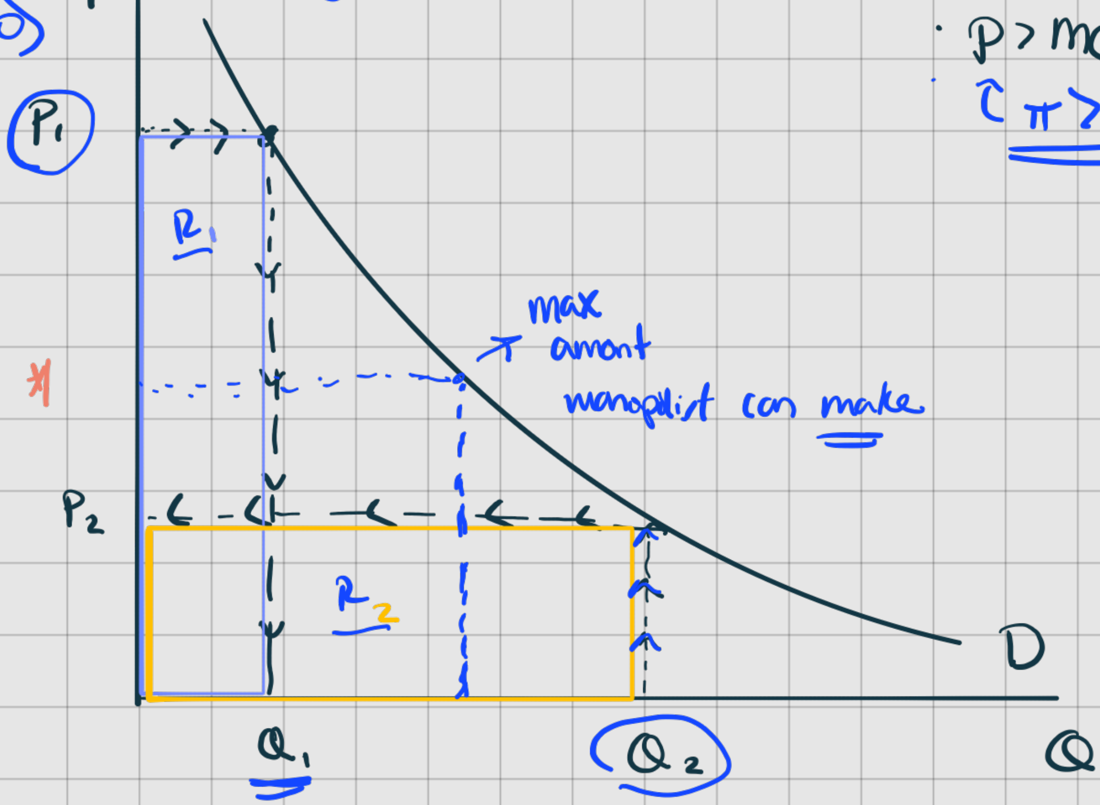
Price discrimation $(P = WTP)$ →

- set P or Q
- one firm
- non-linear rev ← Price changes w Q
- $P > MC$
- $(\pi > 0$

if set Q

if set P

* constrained by demand *



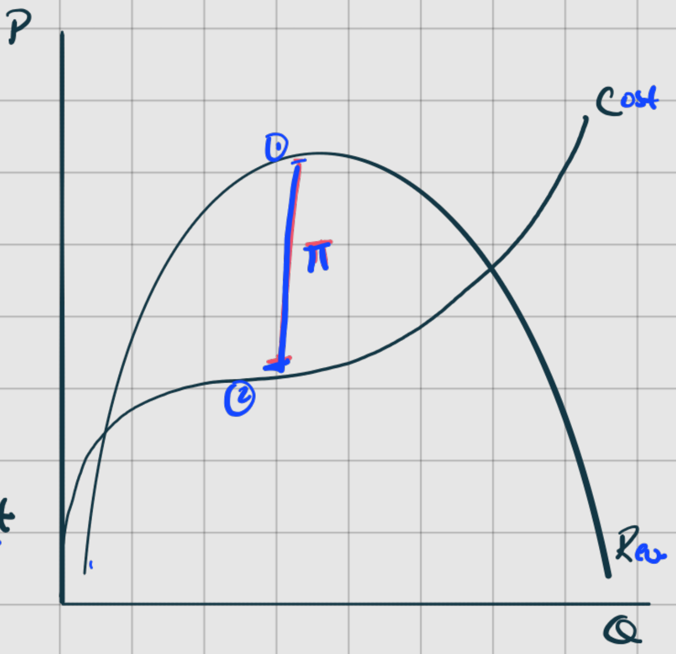
Monopolist Problem (max π)

$$\max_Q \pi \quad R(Q) - c(Q)$$

$$= \underline{P(Q) \times Q} - \underline{c(Q)}$$

$$\text{FOC}: \frac{\partial \pi}{\partial Q} = \frac{\partial R}{\partial Q} - \frac{\partial c}{\partial Q} \boxed{= 0}$$

Marginal Revenue ↗ ↖ Marginal Cost



$\therefore$ we want $\underline{Q^m}$ s.t $\boxed{MR = MC}$ * optimality Condition *

$$\frac{\partial(f(x)g(x))}{\partial x} = f'g + g'f$$ ① ②

$$\hookrightarrow MR(Q) = \frac{\partial[\overset{f(x)}{P(Q)} \times \overset{g(x)}{Q}]}{\partial Q}$$

$$= \overset{f'}{\frac{\partial P(Q)}{\partial Q}} \overset{g}{Q} \frac{P(Q)}{P(Q)} + \overset{f}{P(Q)}\overset{g'}{(1)}$$

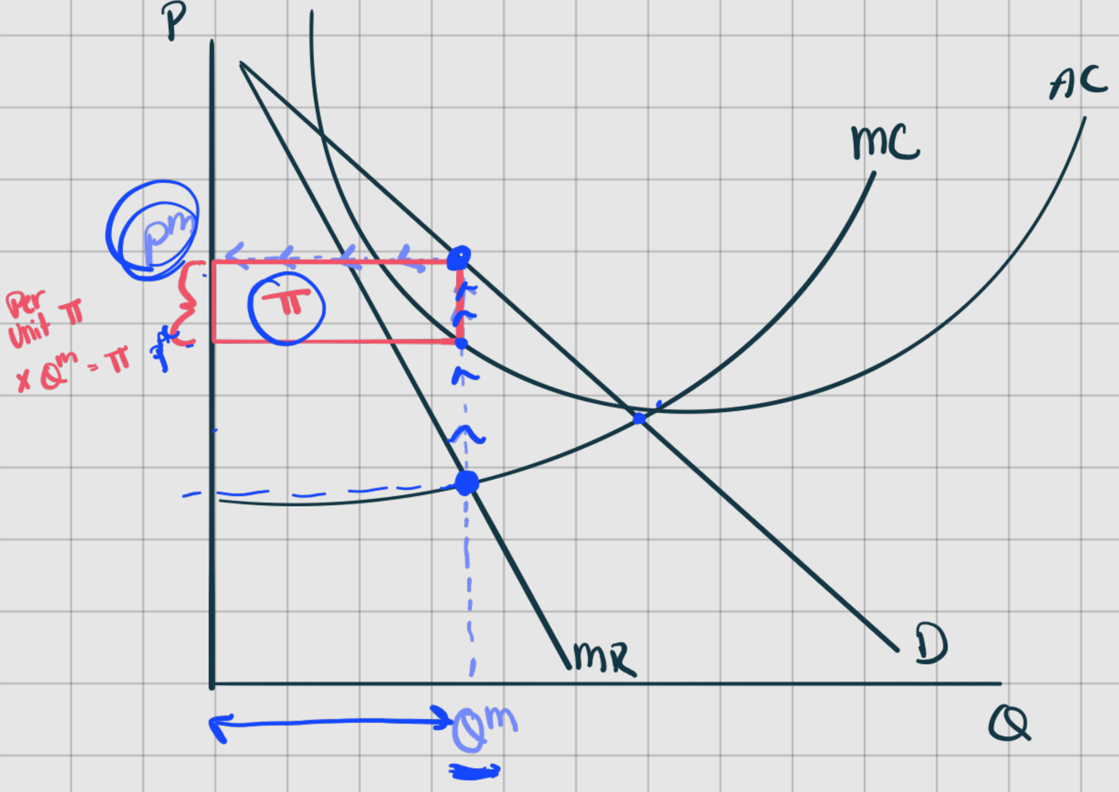
$$= P\left(\frac{1}{\varepsilon} + 1\right) = P\overbrace{\left(1 - \frac{1}{|\varepsilon|}\right)}^{<1} = MR$$

$MR = MC$

$$\therefore MC = P\left(1 - \frac{1}{|\varepsilon|}\right)$$

$\Rightarrow MC < P$ * (+) π *

· Linear example



$\hookrightarrow P = a - bQ$
$C(Q) = cQ$

$$> \max \underline{\pi(Q)} = \overset{TR}{P(Q)Q} - \overset{TC}{C(Q)}$$

$$(a - bQ)Q - cQ$$

$$aQ - bQ^2$$

$$\frac{\partial \pi}{\partial Q} = \overset{MR}{a - 2bQ} \overset{MC}{- c} = 0$$

$$a - c = 2bQ$$

$$\boxed{Q^m = \frac{a - c}{2b}}$$

→ Q^m into demand curve:

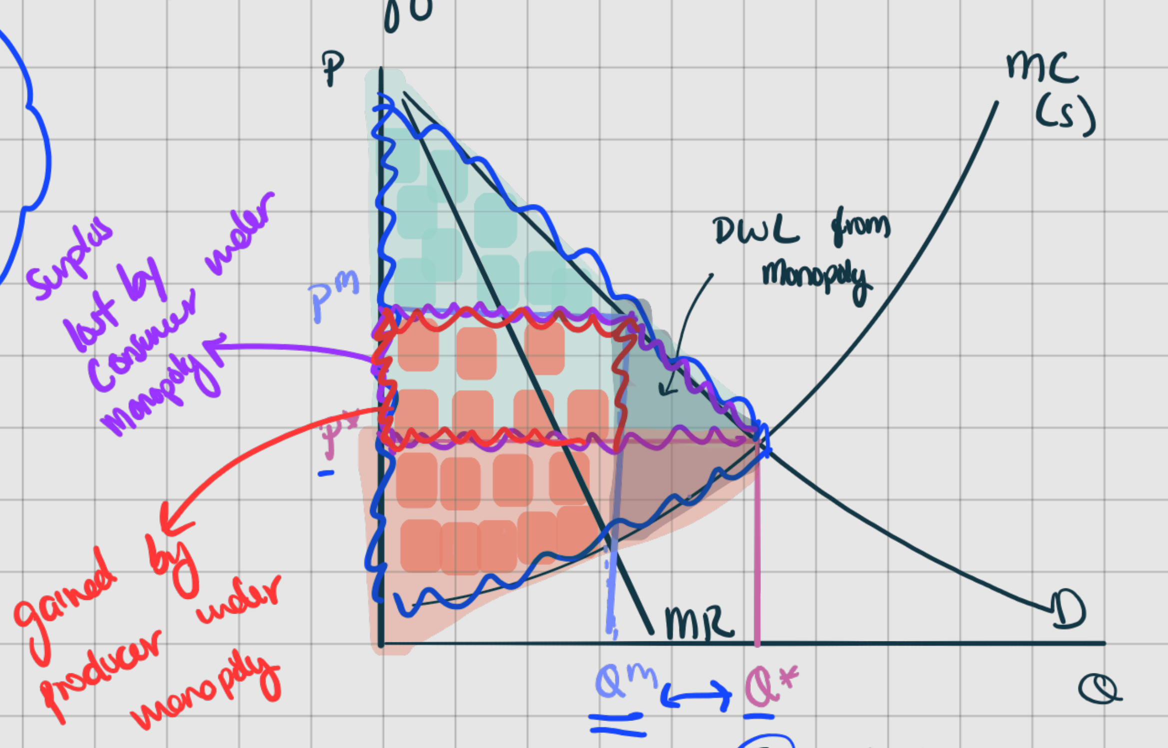
$$P^m = a - \cancel{b}\left(\frac{a-c}{2\cancel{b}}\right) = \frac{2a-(a-c)}{2} = \boxed{\frac{a+c}{2}}$$

• Inefficiencies of monopoly

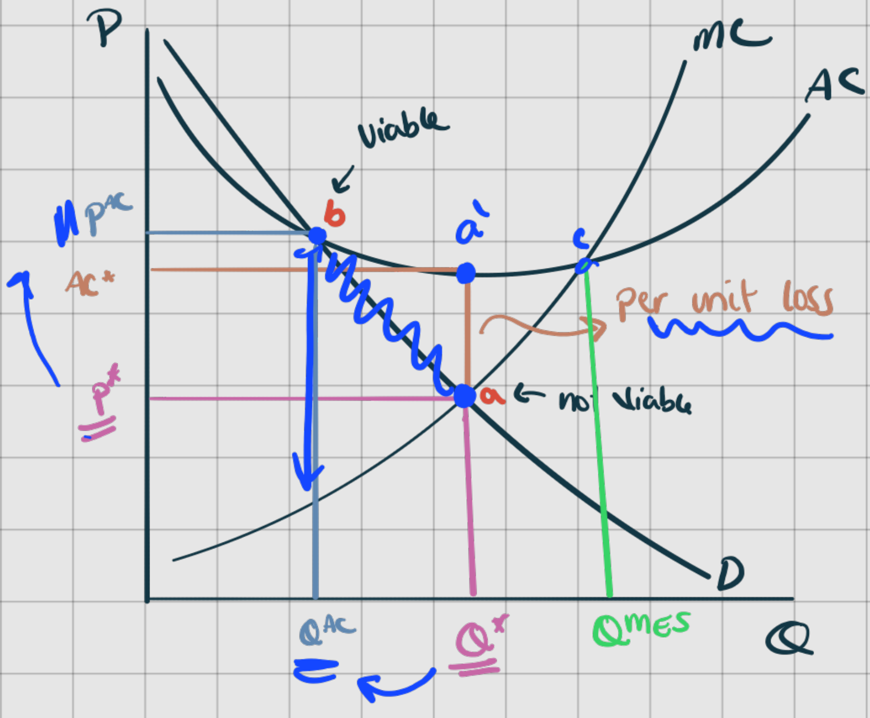
↳ $P > MC$ ∴ consumers worse off

Total economic surplus under P.C is the max

under monopoly
$P^m > P^*$
$Q^m < Q^*$



• natural monopolies (govt, exclusive rights, reliable) → FC is high, MC is small << FC



→ in PC optimal not possible b/c $P < AC$

∴ need to charge $> P^*$

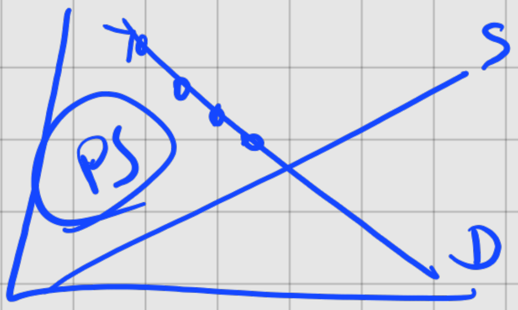
↳ Leads to natural monopoly

$P^{AC} > P^*$

$Q^{AC} > Q^*$

↳ minimum efficient scale (MES) - level of Q that min AC = MC

- Taxes : MC + t ∴ P ↑
- Price discrimination (1) First order - charge WTP ∴ CS = 0
  (2) Second order - charge diff price for different groups (↓ CS*)
- Tariff : P = MC & $\boxed{\pi \text{ from entry fee}}$



(Q1) $P = a - bQ$ $\boxed{P(Q)}$ 2,3,4

$MC = Q$

1) Revenue $= P \cdot Q = (a - bQ)Q$ ✓

2) $MR = \frac{\partial R}{\partial Q} = a - 2bQ$ ✓

Plug into demand curve

$\frac{(1+2b)a - ba}{1+2b}$

3) π max Q^m

$\boxed{MR = MC}$

$a - 2bQ = Q$

$a = Q(1+2b)$

$\boxed{Q^m = \frac{a}{1+2b}}$

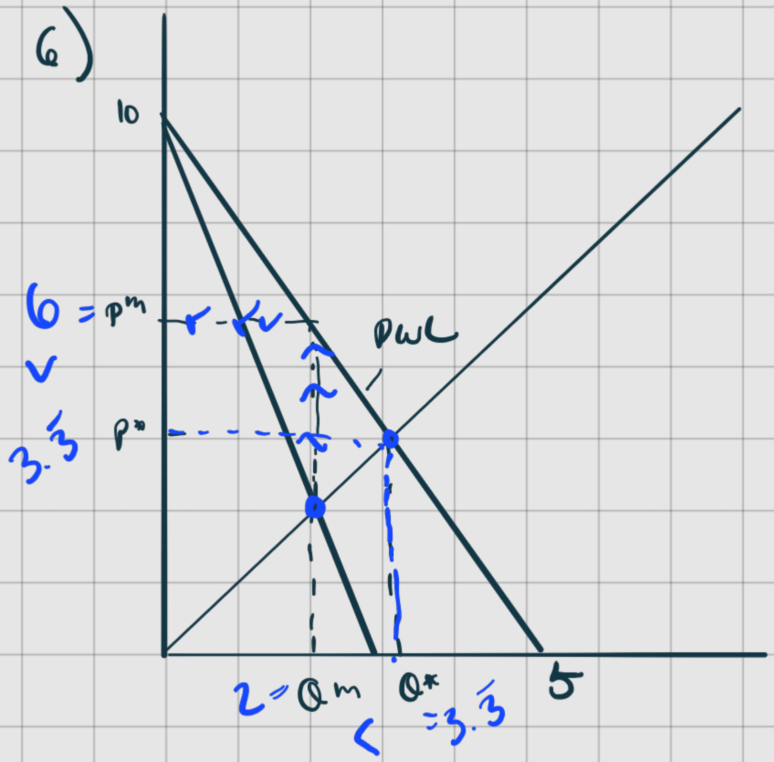
4) $P^m = a - b\left(\frac{a}{1+2b}\right)$

$= \frac{a + ab}{1+2b}$

5) $A = 10$ $Q^m = 2$

$B = 2$ $P^m = 6$

6)



7) P^*, Q^* ?

$MC = P$

$D = S$

$10 - 2Q = Q$

$Q = \frac{10}{3}$ ~ 3.33

$P = \frac{10}{3}$

(Q2) $Q^d = 500 - P \Rightarrow P = 500 - Q$

$TC = 4Q^2$

1) Q^m P^m ?

$MR = MC$

$500Q - Q^2$

$$\frac{\partial((500-Q)(Q))}{\partial Q} = \frac{\partial(4Q^2)}{\partial Q}$$

$$500 - 2Q = 8Q$$

$$\boxed{Q^m = 50} \rightarrow P^m = 500 - Q^m \; \boxed{= 450}$$

2) AC @ eqm Q

$\frac{TC}{Q} =$

$$AC = 4Q$$
$$= 4(50)$$
$$\boxed{= 200}$$

3) π^m ?

$$\pi = TR - TC$$
$$= (500 - Q)Q - 4Q^2$$
$$= 500\overset{50}{Q} - 5\overset{50}{Q}^2$$
$$= 12\,500$$

OR

For each unit / per unit π

π/unit $\Rightarrow P - AC$

$= 450 - 200$

$= 250$

↳ for all Q $(Q \times 250)$ → # that we are producing

$= 12,500$

(Q3) $Q^d = 200 - P \Rightarrow P = 200 - Q$

$(0 < mc < 200) \Rightarrow$

Q^m & p^m ? How do they change we mc

$TR = (200 - Q)Q \Rightarrow MR = 200 - 2Q$

$TC = c \times Q \Rightarrow mc = c$

$MR = MR$

$200 - 2Q = c$

$\downarrow Q^m = 100 - \frac{c}{2} \uparrow \quad \uparrow P^m = 100 + \frac{c}{2} \uparrow$

⇒ the higher the mc, ↓ Q^m

↑ p^m

$0 \Rightarrow 100 - \frac{0}{2} = 100$

Q^m

$p^m = 100 + \frac{0}{2} = 100$

$200 \Rightarrow 100 - \frac{200}{2} = 0$

$100 + \frac{200}{2} = 1200$

(Q4) $\boxed{\begin{array}{l}\varepsilon = -2 \\ P^m = 8\end{array}}$ $\underline{mc} = ?$

special demand

$$Q^d = KP^r \Rightarrow P = \left(\frac{Q}{K}\right)^{1/r}$$

$$TR = \left(\frac{Q}{K}\right)^{1/r} \cdot Q = \frac{1}{K}^{1/r} \cdot Q^{\frac{1}{r}+1}$$

Showed in lecture

$$MR = \left(\frac{1}{K}\right)^{1/r} \left(\frac{1}{r} + 1\right) Q^{\frac{1}{r}} \Rightarrow MR = P\left(\frac{1}{\underset{\varepsilon}{r}} + 1\right)$$

$$= 8\left(\frac{-1}{2} + 1\right)$$

$$= 4$$

$$\therefore \underline{MR = 4} \boxed{= mc}$$