Midterm this Week! (Mod. 1 - 4)

Technology of Production

- Production: transform inputs into set of outputs

labour, kapital, food → Inputs (factors of production) [technology] → output (G & S produced)

- Good & services

- technology: determines the quantity of output that is feasible given inputs ← B.C

output

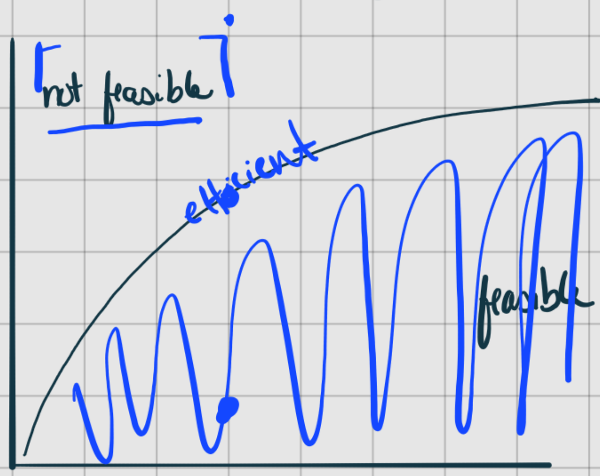
$Q = 2L$

$L = 2$

$Q = 2(2) = 4$

$Q = 1$

$Q = 10$



input

production constraint
↳ divides what is possible & not

→ Production function
↳ max level of output for each level of input.

What is the production set?

output ↓ ← anything

K, L $\quad q = f(L, K)$

↑ examples of functional forms for production function

$q = f(L,K) = L + K$

$q = f(L,K) = AL^{\alpha} K^{\beta}$

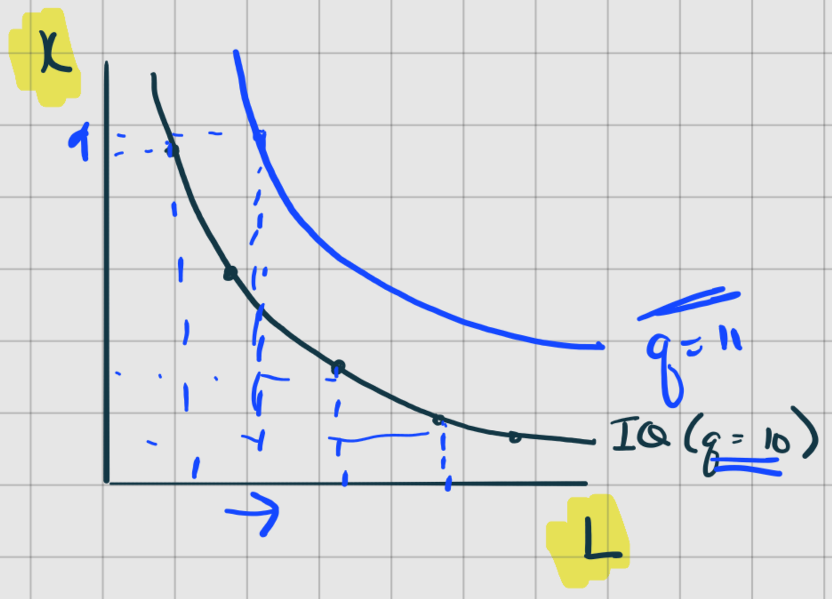
$q = f(L,K) = \sqrt{L}$

⋮

Utility → IDC all the same utility

· **Isoquant**: all combinations of inputs that produce a constant level of output

what is similar to this that we have already looked at?



all them quantity of output

examples: $q = \min\{L, K\}$ fixed proportions / complements

$q = L + K$ substitutes

$\alpha \ln L + B \ln K$ ← $q = A L^{\alpha} K^{\beta}$ Cobb-douglas

* a monotonic transformation of $f(L,K)$ does not give same tech!

↳ not the same tech prod. after transformation

Assumption:

1) monotonic - more inputs more outputs

2) Convexity - average produce more than extremes.

- Marginal Product of . . . . . $\left(\frac{\partial f(L,K)}{\partial} / \partial \ldots\right)$

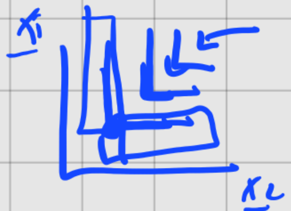
↗ 1 unit Δ in input causes what for outputs

derivative

$$MP_L = \frac{\partial f(L,K)}{\partial L} \qquad MP_K = \frac{\partial f(L,K)}{\partial K}$$

↳ how much output increases if you increase usage of L or K holding other constant

perf comp.



ex: $q = f(L,K) = \boxed{\min}\{L, K/2\}$

* function not differentiable everywhere... specifically at kink *

$$q = \min\{L, K/2\} = \begin{cases} L & \text{if } L < \frac{K}{2} \\ K/2 & \text{if } L > \frac{K}{2} \\ L & \text{if } \boxed{L = \frac{K}{2}} \end{cases} \Rightarrow MP_L = \begin{cases} 1 \\ 0 \\ \cancel{\exists} \end{cases} \; + \; MP_K = \begin{cases} 0 \\ 1/2 \\ \cancel{\exists} \end{cases}$$

- Average Product ... (Per ___ output, $\frac{f(L,K)}{L \text{ or } K}$)

$$AP_L = \frac{f(L,K)}{L} \qquad AP_K = \frac{f(L,K)}{K}$$

division

↳ Relationship b/w MP + AP

AP < MP

0 - 1 - 2 - 3

each addition is adding more + more

ex: $L = 100 \quad q = 1000$

$MP_L = 50 \therefore L = 101 \rightarrow 1050$

weigh down average

$$AP_L = \frac{1000}{100} = \boxed{10} \qquad AP_L = \frac{1050}{101} > \boxed{10}$$

more than 10

if $AP_L < MP_L \therefore \uparrow L$ will increase AP_L

* gains from L pull up AP *

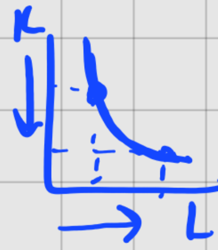
↳ AP_L increasing in L

if $AP_L > MP_L$ ∴ ↑ L will decrease AP_L

↳ AP_L decreasing in L   1 - 9   10 → 11
0   10   10   9

· Technical Rat of Sub (TRS)

$TRS = -\frac{MP_L}{MP_K}$ how much can you reduce K $(-\Delta K)$ & ↑ L (ΔL) to reach the same production level



ex: $q = 2L + 3K$ $TRS = -\frac{MP_L}{MP_L} = \boxed{-\frac{2}{3}}$

$q = L^{1/2} K^2$ $TRS = -\frac{\frac{1}{2} K^2 L^{-\frac{1}{2}}}{2 L^{1/2} K}$

$\boxed{= -\frac{1}{4} \frac{K}{L}}$

9 → 10 → 11 → 12
100   50   10

· Diminishing marginal Product / Returns

· more of single input ↑ output c̄ decreasing rate

$\left[\frac{\partial MP_L}{\partial L} < 0 \right]$ ∴ MP_L decreasing with L

$\frac{\partial MP_K}{\partial K} > 0$

↳ increasing in K
decreasing as ↓K

* can be diminishing in one input & not the other! *

· Returns to Scale (RTS)

what happens to my output if I change all inputs?

1) Constant Returns to scale (CRS) $f(tL, tK) = t f(L, K)$
2) Increasing Returns to Scale (↑RS) $f(tL, tK) > t f(L, K)$ specialization
3) Decreasing Returns to Scale (↓RS) $f(tL, tK) < t f(L, K)$ search cost

ex : $q = f(L,K) = LK^2$

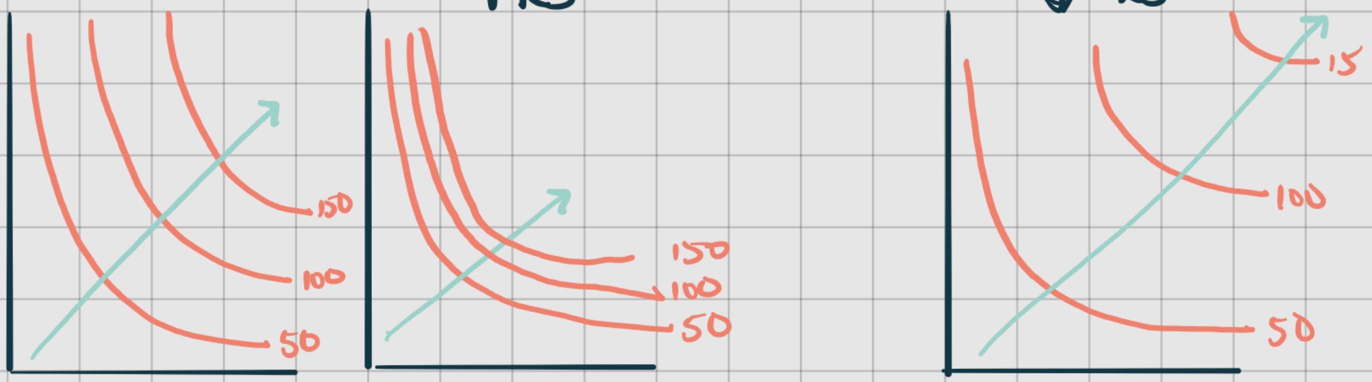
$\left| tL(tK)^2 \right| > t(LK^2)$

tLt^2K^2

$t^3 LK^2$

$\therefore \uparrow RS$

CRS ↑RS ↓RS



* ↑ output more than Δ in input *

* ↑ output less than Δ in output *

↳ RTS are local - answer Δ w̄ different t

ex: $f(L,K) = AL^{\alpha}K^{\beta}$

$A(tL)^{\alpha}(tK)^{\beta}$

$At^{\alpha}L^{\alpha}t^{\beta}K^{\beta}$

$[t^{\alpha+\beta}] A\cancel{L^{\alpha}K^{\beta}}$

$[t] A\cancel{L^{\alpha}K^{\beta}}$

all ~~so~~ ⇒ q

>1

$t^{\alpha+\beta}$ vs t

$0 < \alpha < 1$
β

CRS	if	$\alpha + \beta = 1$ (0.5 0.5)	
↑RS	if	$\alpha + \beta > 1$	$0.6 + 0.6 = 1.2$
↓RS	if	$\alpha + \beta < 1$	$0.6 + 0.3 = 0.9$

$\alpha = \frac{1}{2}$

$\beta = \frac{1}{2}$

0

· Short Run vs Long Run

· long Run = all factors can be adjusted

· Short Run = at least one factor cannot be adjusted (ie $\bar{K}$ ∴ $f(L,\bar{K})$)

Strictly

ex: $q = L + K$

SR: $K = \bar{K} = 50$ ∴ $q = L + 50$ * only L can change *

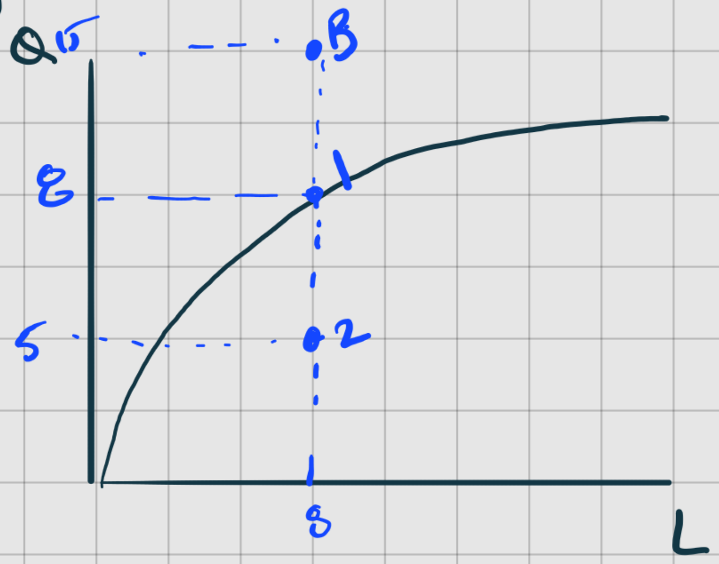
when do firm continue to operate .

Q1: $Q = 4L^{1/3}$ $\qquad 8^{1/3} = 2 \cdot 4 = 8$ 1, 2, 3

1) technically efficient $\qquad L=8 \Rightarrow Q=8$

2) technically inefficient $\qquad L=8 \Rightarrow Q=5$

3) technically unattainable $\qquad L=8 \Rightarrow Q=15$

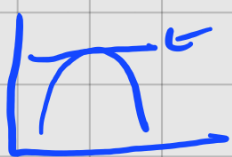


Q2: $Q = 4L + 12L^2 - 6L^3$

1) $AP_L = \dfrac{4L + 12L^2 - 6L^3}{L} = 4 + 12L - 6L^2$

2) $MP_L = \dfrac{\partial Q}{\partial L} = 4 + 24L - 18L^2$

3) L^* for max AP_L



derivative is 0
is slope is 0

max $AP_L \Rightarrow \dfrac{\partial AP_L}{\partial L} = 0$

$12 - 12L = 0$

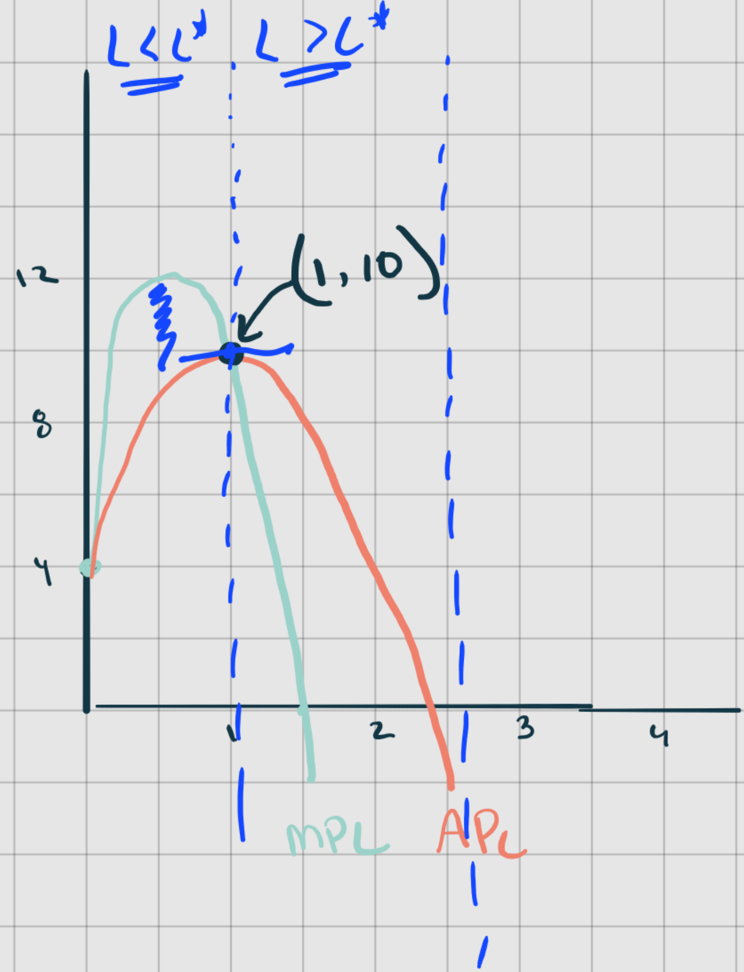
$\boxed{L^* = 1}$

4) if $L > L^*$ $\therefore$ $\underline{MP_L < AP_L}$ ← ↓ average

$L < L^*$ $\therefore$ $MP_L > AP_L$ ← each gain ↑ Average

$L = L^*$ $\therefore$ $MP_L = AP_L$

5)



Q3: $\underline{Q = AL^{\alpha}K^{\beta}}$

1 → 2 → 3

1) $MP_L = \frac{\partial Q}{\partial L} = [A\alpha L^{\alpha-1}K^{\beta}]$

$\frac{\partial MP_L}{\partial L} = \alpha A(\alpha-1)L^{\alpha-2}K^{\beta} < 0$ $\therefore$ $\underline{\text{diminishing}}$

+ + − + +

↑ MP_L ↓ in L

$$MP_K = AL^{\alpha} B K^{B-1}$$

$$\frac{\partial MP_K}{\partial K} = AL^{\alpha} B(B-1) K^{B-2} < 0 \quad \boxed{\therefore \text{ dim.}}$$

$+ \quad + \quad + \quad (-) \quad +$

2) $AP_L = \frac{AL^{\alpha}K^{B}}{L} = AL^{\alpha-1}K^{B}$

$AP_K = . \qquad = AL^{\alpha}K^{B-1}$

3) $|TRS| = \frac{MP_L}{MP_K} = \frac{A\alpha L^{\alpha-1}K^{B}}{ABL^{\alpha}B^{B-1}} \left[= \frac{\alpha}{B}\frac{K}{L} \right]$

when is TRS diminshing

$$\frac{\partial TRS}{\partial L} = \left(\frac{\alpha K}{B}\right) \cdot (-1) \cdot \frac{1}{L^2} < 0 \quad \therefore \text{ dim}$$

diminish as $\uparrow L$

$$\left[\frac{\partial TRS}{\partial K} = \left(\frac{\alpha}{BL}\right) \cdot (1) \right. \quad > 0 \quad \therefore \text{ increasing}$$

diminish as $\downarrow K$

* altumatively can look @ what happens to TRS if *
K or L Δ

$$TRS = \left[\frac{\alpha}{B}\frac{K}{L}\right]$$

4) $t\,AL^{\alpha}K^{B} \qquad A(tL)^{\alpha}(tK)^{B}$

$t^{\alpha+B} AL^{\alpha}K^{B}$

$\therefore$ if $a+b=1$ CRS

$a + b > 1$ $\uparrow RS$

$a + b < 1$ $\downarrow RS$