

# Demand:

1, 3, 5

Demand function - indicates optimal choice for a given set of P's + m  
 $x^*(P_1, P_2, m)$  - also from preferences

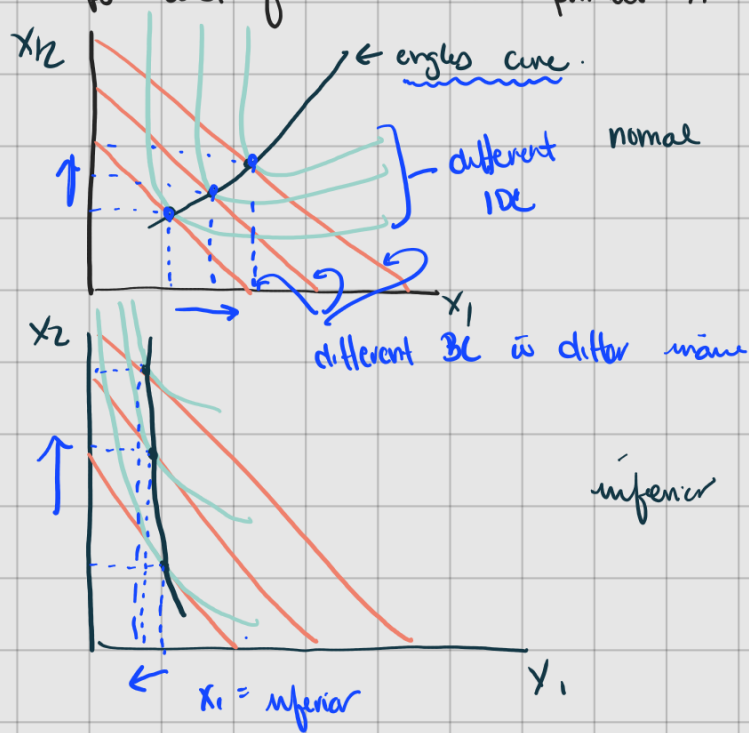
↳ how does individual demand change:  $x_1^* = \frac{a}{a+b} \frac{m}{P_1} P_2$

(1) ↑ in m:

$\frac{\partial x_1^*}{\partial m}$	$\begin{cases} > 0 \\ < 0 \end{cases}$	normal good (↑ consumption as ↑ m)	↳ Vaccation
		inferior Good (↓ consumption as ↑ m)	↳ store brand

\* engel's curve: maps level of income to optimal  $x_1^* + x_2^*$

how does consumption of  $x_1$  change w/ m



$$x_1^* = \frac{a}{a+b} \frac{m}{P_1} P_2$$

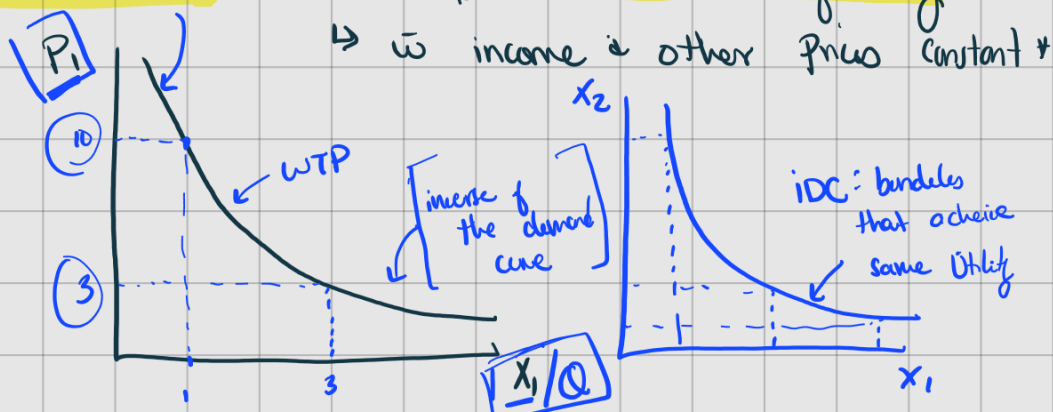
(2) ↑ in  $P_1$

$\frac{\partial x_1^*}{\partial P_1}$	$\begin{cases} < 0 \\ > 0 \end{cases}$	ordinary Good (↓ consumption as ↑ $P_1$ )	↳ gas
		given Good (↑ consumption as ↑ $P_1$ )	↳ collectable

\* demand curve: relationship b/w price + quantity

how does consumption of  $x_1$  Δ w/  $P_1$

\*  $P_2, m$ , are constant



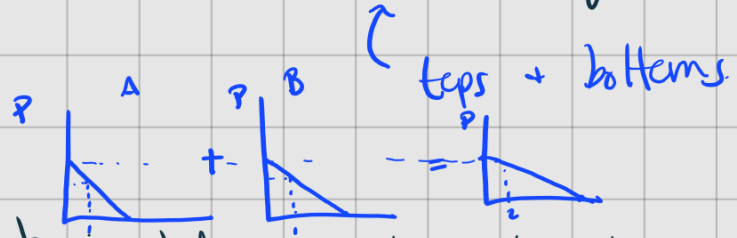
↳ w/ income & other prices constant

(3) ↑ in  $P_2$ :  $X_1^* = \frac{g}{a+b} \frac{m}{P_1} P_2$  ↙ OJ for apple juice

$\frac{\partial X_1^*}{\partial P_2} > 0$  Gross substitutes (↑ consumption of  $P_2$  ↑)

$\frac{\partial P_2}{\partial P_2} < 0$  Gross complements (↓ consumption of  $P_2$  ↑)

how does consumption of  $X_1^*$  as the price of  $X_2$  changes

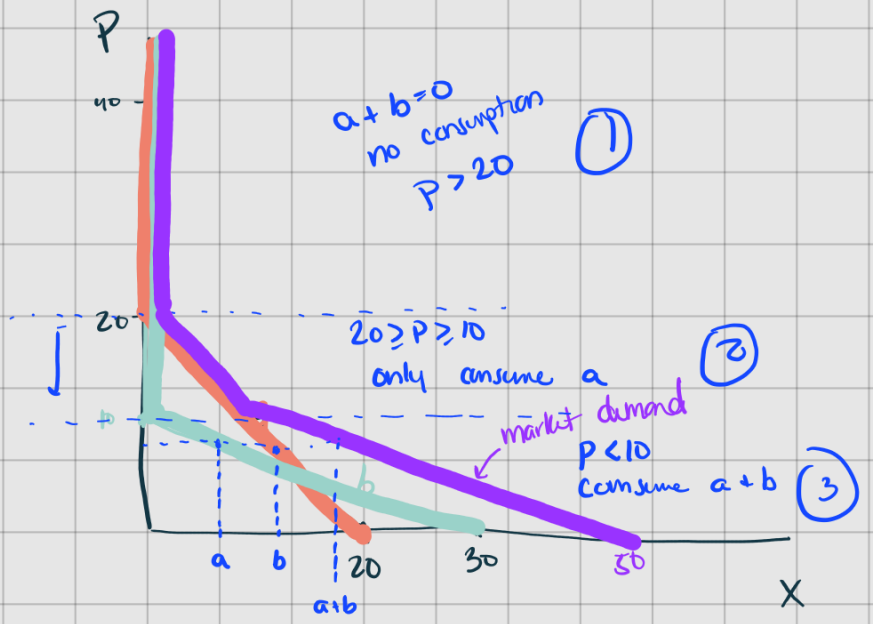


Market Demand = horizontal summation of individual demand

ie  $\left[ \begin{array}{l} 100 \text{ individuals} \\ X_i^* = \frac{g}{a+b} \frac{m}{P_1} \end{array} \right] \rightarrow X = 100 \cdot X_i^* = 100 \left( \frac{g}{a+b} \frac{m}{P_1} \right)$

for each price how much  $X$  is consumed

ie:  $a: X = 20 - P$   
 $b: X = 30 - 3P$



$X=0$   $P=?$   
 $0 = 20 - P$   
 $P = 20$   
 $P=0$   $X=?$   
 $X = 20 - 0$   
 $X = 20$

$X=0$   $P=?$   
 $0 = 30 - 3P$   
 $P = 10$   
 $P=0$   $X=?$   
 $X = 30 - 3(0)$   
 $X = 30$

$$X = X_a + X_b = \begin{cases} 0 & \text{if } P > 20 \\ 20 - p & \text{if } 10 < P < 20 \\ 50 - 4p & \text{if } P < 10 \end{cases}$$

$$a + b = 20 - P + 30 - 3P = 50 - 4P$$

↳ General case

$i = 1, 2, 3, 4$

consumers:  $i \in \{1, 2, \dots, N\}$

demand:  $x^i(P_1, P_2, m_i)$

same person

Market demand =  $X = \sum_{i=1}^{i=N} x^i(P_1, P_2, m_i)$

*total # of consumers* (pointing to  $i=N$ )  
*starting at 1* (pointing to  $i=1$ )

individual ppl are price takers

$X = \sum_{i=1}^{i=2} x^i(P_1, P_2, m_i)$

+  $\sum_{i=3} x^i(P_1, P_2, m_i)$

WTP

discrete goods + Reservation Price

$U(\text{Toyota}, m - P_T)$

either 1/0

$U = (\text{car}, \$)$

$(\text{Car}, m - P)$     $(\text{No.}, m)$

compare the utility gain from having to the price of car

$\Rightarrow$  if  $T=1 : U = (1, m - P_T)$   
 $T=0 : U = (0, m)$

$\Rightarrow$  if  $\frac{\partial U}{\partial T} \geq 0 \therefore U(1, m) > U(0, m)$

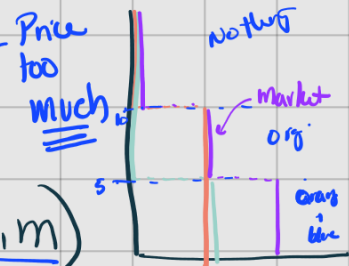
for high enough  $P_T$

$U(1, m - P_T) < U(0, m)$

\* if Price below \* then purchase of not, than do not

Reservation Price

WTP anything below the RP



Elasticity - measure of responsiveness of demand to price

Own  
cross  
income

$$|E| = \frac{P}{X} \frac{\partial X}{\partial P} \quad \text{aka} \quad \frac{\frac{\Delta X}{X}}{\frac{\Delta P}{P}} \quad \frac{\frac{DX}{X}}{\text{rate of } \Delta \text{ of } P}$$

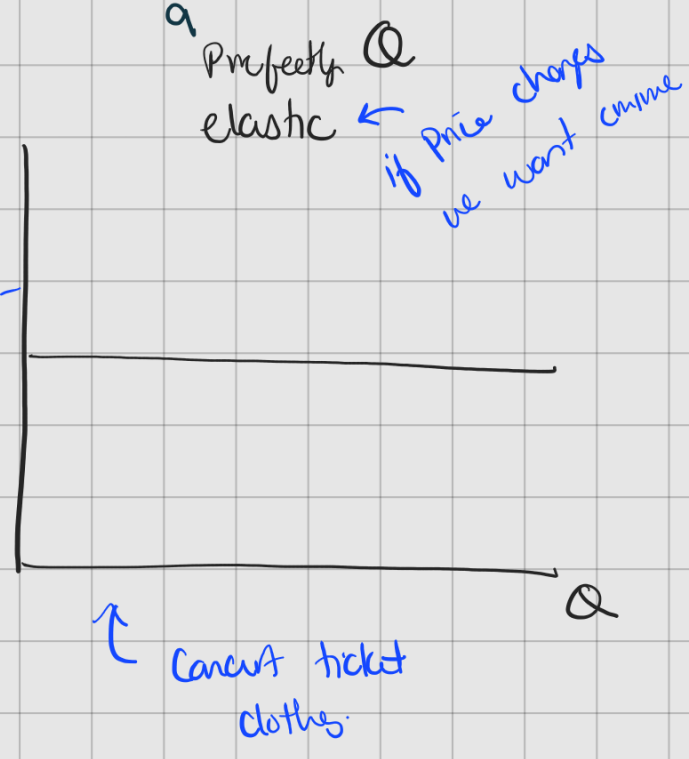
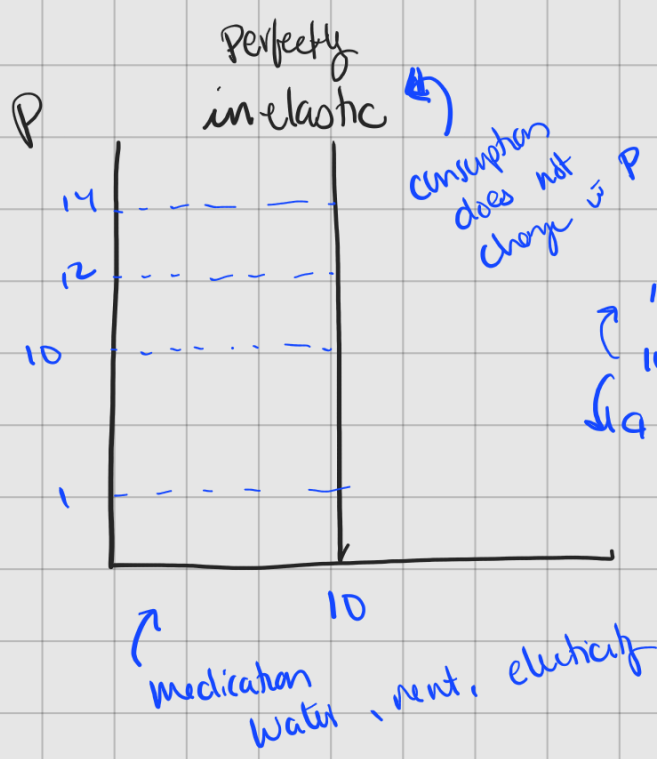
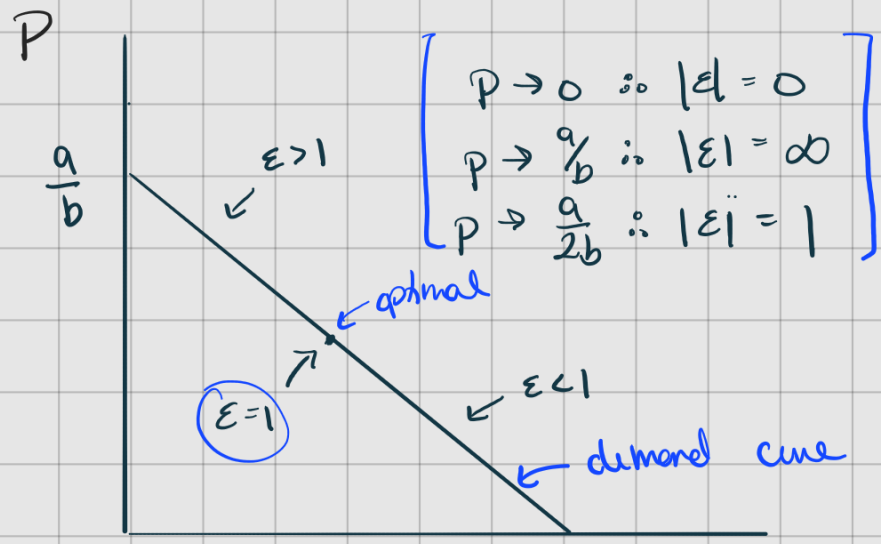
$\frac{DX}{DP}$  = derivative  
rate of  $\Delta$  of X

$|E| > 1$  elastic demand (demand  $\Delta$  w  $P$ )  
 $< 1$  Inelastic demand (demand  $\downarrow$  w  $P$ )

ex:  $q = a - bp$

demand  
function

$$E = \frac{P}{q} \frac{\partial q}{\partial P} = \frac{P}{a - bP} (-b) = \frac{-Pb}{a - bP}$$



(Q1) 90 consumers  $\left\{ \begin{array}{l} 60 = A \Rightarrow u(x,y) = x^{1/3} y^{1/3} \quad m_a \\ 30 = B \Rightarrow u(x,y) = x^{2/3} y^{1/3} \quad m_b \end{array} \right. \text{CD}$

(#1) demand for A

$$X = \frac{\frac{1}{3} m_a}{\frac{1}{3} + \frac{1}{3}} P_x \quad Y = \frac{1}{2} \frac{m}{P_y}$$

$$X_a = \frac{a}{a+b} \frac{m}{P} = \frac{1}{2} \frac{m_a}{P_x} \quad Y_a = \frac{1}{2} \frac{m_a}{P_y}$$

(#2) demand for B

$$X_b = \frac{a}{a+b} \frac{m}{P} = \frac{2}{3} \frac{m_b}{P_x} \quad Y_b = \frac{1}{3} \frac{m_b}{P_y}$$

$$X = \frac{\frac{2}{3} m}{\frac{1}{3} + \frac{2}{3}} P_x \quad Y = \frac{1/3}{2 + 1/3} \frac{m}{P_y}$$

(#3) market demand of X

$$X = \left( \frac{1}{2} \frac{m_a}{P_x} \right) 60 + \left( \frac{2}{3} \frac{m_b}{P_x} \right) 30 = 30 \frac{m_a}{P_x} + 20 \frac{m_b}{P_x}$$

# of ppl in B

(#4) Market demand of Y

$$Y = \left( \frac{1}{2} \frac{m_a}{P_y} \right) 60 + \left( \frac{1}{3} \frac{m_b}{P_y} \right) 30 = 30 \frac{m_a}{P_y} + 10 \frac{m_b}{P_y}$$

(#5)

$P_x = 2$
$P_y = 2$
$m_a = 60$
$m_b = 120$

$$X = 30 \left( \frac{60}{2} \right) + 20 \left( \frac{120}{2} \right) = 2100$$

$$Y = 30 \left( \frac{60}{2} \right) + 10 \left( \frac{120}{2} \right) = 1500$$

(Q3) 200 individuals  $\rightarrow \frac{\partial X}{\partial P_x} = -\frac{1}{3} m \left(\frac{1}{P_x^2}\right) \Rightarrow \ominus < 0$  ∴ Ordinary Good  
 less consumption  
 ∴ ↑ P<sub>x</sub>

(#1) all have same demand  $[u = X^{1/3} Y^{2/3}]$  ∴ Market demand of X

$X = \frac{1}{3} \frac{m}{P_x}$   
 ↑ individual

$X = 200 \left(\frac{1}{3} \frac{m}{P_x}\right) \Rightarrow \frac{200 m}{3 P_x}$   
 ↑ Market / total

(#2) own Price elasticity of demand of X, constant or changing?

how does  
 change of  
 P<sub>x</sub> ∴  
 consup of X

$E = \frac{P}{X} \cdot \left[ \frac{\partial X}{\partial P} \right] = \frac{P_x}{\frac{200 m}{3 P_x}} \cdot \left( -\frac{1}{3} \right) \left( \frac{m}{P_x^2} \right) = -1$

$|E| = 1$

(#3) m = 150  
 P<sub>x</sub> = 4  
 P<sub>y</sub> = 3

X = 2500

$-\frac{P_x \left(\frac{200}{3}\right) m}{\frac{200 m}{3} P_x^2} = -1$

(#4)

(05) Adam :  $X_a = \max\{0, 20 - 2p\}$  ← not perfect complements  
Barbara :  $X_b = \max\{0, 30 - 2p\}$

Find market demand  $\downarrow$   $\epsilon$  c  $P=40$

$$P=40 \therefore X_a = \max\{0, 20 - 2(40)\}$$
$$= \max\{0, -60\}$$
$$\boxed{=0}$$

$$X_a^* = 0$$

$$X_b = \max\{0, 30 - 2(40)\}$$
$$= \max\{0, -50\}$$
$$\boxed{=0}$$

$$X_b^* = 0$$

$$\leftarrow + = 0$$

$\Rightarrow \therefore$  Market demand is 0  
 $\epsilon = 0$  (perfectly elastic)