Consumer Choice : Best bundle or set that is available.

* combines both preferences + budget *

↳ Optimal choice : Bundle w the highest Utility among affordable option

? Constrained vs unconstrained optimization?

unconstrained ⇒ only an objective function (ie: What is the best house?) → 20m

constrained ⇒ max objective s.t a constraint (ie: what is the best house I can afford?)

Utility — B.C

↓ 1 - 2m

Optimal choice for Cobb-Douglas (tangency sol^n)

$$u(x,x) = \underbrace{x_1^2 x_2^2}_{\downarrow}$$

· where is affordable set?

↳ what bundle is afforable?

↳ what IDC is the highest

⇒ is that the same as the best we can afford

↳ why is "b" the Best?

method 1 : Substitution

→ ① identify the utility function (objective) ←

② Calculate the $\boxed{MRS = \frac{-MU_1}{MU_2}}$

③ Set MRS to slope of BL $(-P_1/P_2)$ (#1) * optimality condition *

④ identify the budget line (#2)

⑤ Use eq^n #1 + #2 to solve for X_1 + X_2

Solvable b/c : 2 equations + 2 unknows

ex: $u(x_1, x_2) = x_1^a x_2^b$, P_1, P_2, m

(find Utility)

$$\max \; u(x_1, x_2) = x_1^a x_2^b \qquad x_1^2 x_2^4 \qquad x_1^{1/2} x_2^{1/2}$$

[redacted]

(MRS)

$$MU_1 = \frac{\partial u(x_1, x_2)}{\partial x_1} = a x_1^{a-1} x_2^b$$

$$MU_2 = \frac{\partial u(x_1, x_2)}{\partial x_2} = x_1^a b x_2^{b-1}$$

$$MRS = -\frac{MU_1}{MU_2} = -\frac{a x_1^{a-1} x_2^b}{b x_1^a x_2^{b-1}} = \boxed{-\frac{a}{b} \cdot \frac{x_2}{x_1}}$$

$a-1-(a)$ $\quad$ $b-(b+1)$

(MRS = $-P_1/P_2$)

$$-\frac{a x_2}{b x_1} = -\frac{P_1}{P_2} \quad P_2 \; b x_1 \quad P_1 \; x_1 b \Rightarrow \underline{P_2(a x_2) = P_1(b x_1)}$$

* optimality Condition * $\quad$ eq #1

$|MRS| = P_1/P_2$

(BC) $P_1 x_1 + P_2 x_2 = m$ $\quad$ * eq 2 * $\quad$ Solve

(Solve)

from eq 1: $x_2 = \dfrac{b P_1 x_1}{a P_2}$

sub into eq 2: $P_1 x_1 + \cancel{P_2}\left(\dfrac{b P_1 x_1}{a \cancel{P_2}}\right) = m$ $\quad$ 1 equ. 1 unknow

$$P_1 x_1\left(1 + \frac{b}{a}\right) = m \Rightarrow \boxed{x_1 = \frac{m}{\left(1 + \frac{b}{a}\right) P_1}} \Rightarrow x_1 = \frac{m}{P_1 \frac{a+b}{a}}$$

$$= \frac{ma}{P_1(a+b)} = \frac{a}{a+b} \cdot \frac{m}{P_1}$$

$$\frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} \frac{m}{P_1} \qquad \frac{1}{2} \frac{m}{P_1}$$

Solve for x_2: $x_2 = \dfrac{b \cancel{P_1}}{\cancel{a} P_2} \underbrace{\left(\dfrac{\cancel{a}}{a+b}\right)\left(\dfrac{m}{\cancel{P_1}}\right)}_{x_1}$

$$= \boxed{\frac{b}{a+b} \; \frac{m}{P_2}}$$

$u = x_1^a x_2^b$

optimal bundle (x_1^*, x_2^*)

$$\left(\underbrace{\frac{a}{a+b} \frac{m}{P_1}}_{x_1^*}, \underbrace{\frac{b}{a+b} \frac{m}{P_2}}_{x_2^*}\right)$$

[Specific Point]

Method 2: Lagrangean

1) define the problem
2) take derivatives
3) solve for X_1
4) plug into B.C (constraint)
5) solve for X_2^*

- always

1) Define the problem

subject to

$$\max_{x_1 x_2} \; x_1^{\alpha} x_2^{\beta} \quad \text{s.t} \quad P_1 X_1 + P_2 X_2 = m$$

$$\hookrightarrow \; L = \underbrace{x_1^{\alpha} x_2^{\beta}}_{MU} - \lambda \underbrace{(P_1 X_1 + P_2 X_2 - m)}_{BC}$$

2) take derivative $\left(\frac{\partial L}{\partial x_1}, \frac{\partial L}{\partial x_2}, \frac{\partial L}{\partial \lambda}\right)$

$$\frac{\partial L}{\partial x_1} = \alpha X_1^{\alpha-1} X_2^{\beta} - \lambda P_1 = 0$$

$$\frac{\partial L}{\partial x_2} = \beta X_1^{\alpha} X_2^{\beta-1} - \lambda P_2 = 0$$

$$\frac{\partial L}{\partial \lambda} = \underbrace{P_1 X_1 + P_2 X_2 - m = 0}_{B.C}$$

3) Solve for X_1

$$\lambda = \frac{\alpha X_1^{\alpha-1} X_2^{\beta}}{P_1} = \frac{\beta X_1^{\alpha} X_2^{\beta-1}}{P_2} = \lambda$$

$\beta - \beta + 1 = 1$ $\qquad$ $\alpha - \alpha + 1 = 1$

$$\frac{P_2}{P_1} \cdot \frac{\alpha X_2^{\beta}}{\beta X_2^{\beta-1}} = \frac{X_1^{\alpha}}{X_1^{\alpha-1}}$$

$$\boxed{\frac{P_2 \alpha}{P_1 \beta} \cdot X_2 = X_1}$$

4) Plug into B.C

$$\cancel{P_1}\left(\frac{P_2 \alpha \; X_2}{\cancel{P_1} \beta}\right) + P_2 X_2 = m$$

$$X_2\left(\frac{\alpha}{\beta} P_2 + P_2\right) = m$$

$$X_2 = \frac{m}{\frac{\alpha}{\beta} P_2 + P_2 \left(\frac{\beta}{\beta}\right)}$$

$$= \frac{m}{\frac{\alpha P_2 + \beta P_2}{\beta}} = \frac{\beta \cdot m}{\alpha P_2 + \beta P_2}$$

$$\boxed{X_2^* = \frac{\beta}{\alpha + \beta} \cdot \frac{m}{P_2}}$$

5) Solve for X_1^*

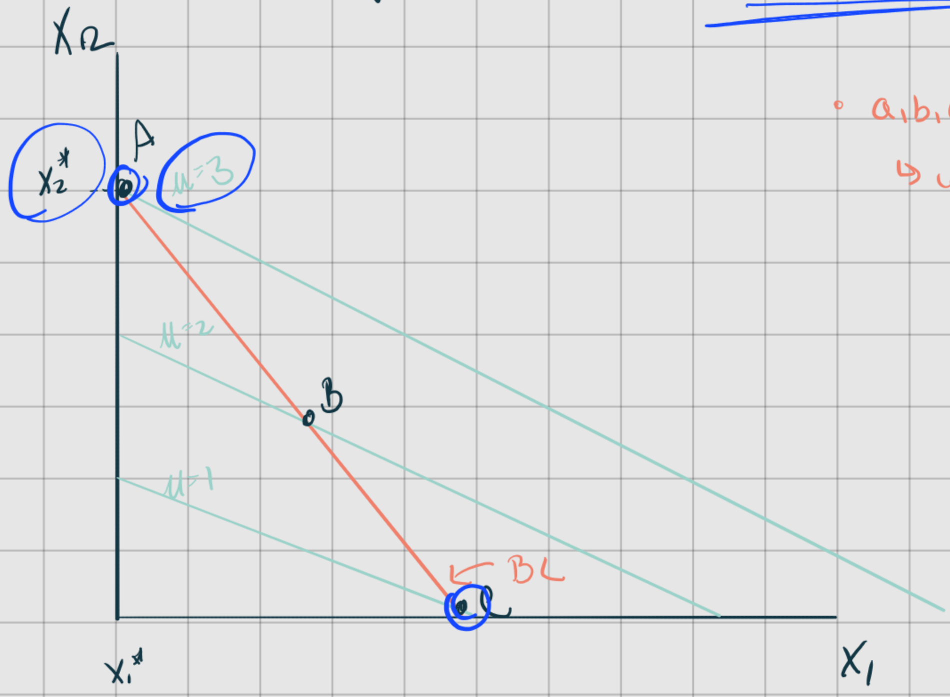
$$\frac{\cancel{P_2} \alpha}{P_1 \cancel{\beta}} \cdot \frac{\cancel{\beta}}{(\alpha + \beta)} \cdot \frac{m}{\cancel{P_2}} = X_1$$

$$\frac{\alpha}{\alpha + \beta} \cdot \frac{m}{P_1} = X_1^*$$

$\Rightarrow \therefore$ optimal bundle

$$(x_1^*, x_2^*) = \left(\frac{\alpha}{\alpha+\beta} \cdot \frac{m}{P_1}, \frac{\beta}{\alpha+\beta} \cdot \frac{m}{P_2}\right)$$

Optimal choice for perfect substitutes (corner solution)



- a, b, c are all on same BC
  - ↳ which is Best?

Steps:

1) define the problem
2) calculate MRS
3) compare MRS to price ratio
4) determine optimal x_1^* + x_2^*

$$\boxed{u(x_1, x_2) = \alpha x_1 + \beta x_2} \qquad P_1 \quad P_2 \quad m$$

1) Define the problem

$$\max_{x_1, x_2} \ \alpha x_1 + \beta x_2 \quad \text{s.t.} \quad P_1 x_1 + P_2 x_2 = m$$

2) MU_1 + MU_2 ⇒ MRS

$$\frac{\partial u}{\partial x_1} = MU_1 = \alpha$$

$$\frac{\partial u}{\partial x_2} = MU_2 = \beta$$

$$\hookrightarrow \boxed{MRS = \frac{-MU_1}{MU_2} = \frac{-\alpha}{\beta}}$$

3) $|MRS|$ vs Price Ratio

$$\frac{\alpha}{\beta} \ \text{vs} \ \frac{P_1}{P_2}$$

$$\boxed{\frac{\alpha}{P_1}} > \frac{\beta}{P_2}$$

→ comparing how you value the 2 goods

↳ which is a better deal

* if given values can find *

optimal from comparing

$MRS \sim P_1/P_2$

* comparing 2 constants *

⇒ ∴ optimal (x_1^*, x_2^*) (depends on comparison)

$$x_1^* = \begin{cases} \frac{m}{P_1} & \text{if } \frac{\alpha}{\beta} > \frac{P_1}{P_2} \\ 0 & \text{if } \frac{\alpha}{\beta} < \frac{P_1}{P_2} \\ x_1^* \in \left[0, \frac{m}{P_1}\right] & \text{if } \frac{\alpha}{\beta} = \frac{P_1}{P_2} \end{cases} \qquad x_2^* = \begin{cases} 0 & \text{if } \frac{\alpha}{\beta} > \frac{P_1}{P_2} \\ \frac{m}{P_2} & \text{if } \frac{\alpha}{\beta} < \frac{P_1}{P_2} \\ x_2^* \in \left[0, \frac{m}{P_2}\right] & \text{if } \frac{\alpha}{\beta} = \frac{P_1}{P_2} \end{cases}$$

Any point btw 0 + $\frac{m}{P_1}$

$A = \frac{\alpha}{\beta} > \frac{P_1}{P_2}$ $\quad \left(\frac{m}{P_1}, 0\right)$

$B = \frac{\alpha}{\beta} < \frac{P_1}{P_2}$ $\quad \left(0, \frac{m}{P_2}\right)$

$C = \frac{\alpha}{\beta} = \frac{P_1}{P_2}$ $\quad$ Any value

$\alpha = 5$ $\quad P_1 = 2$

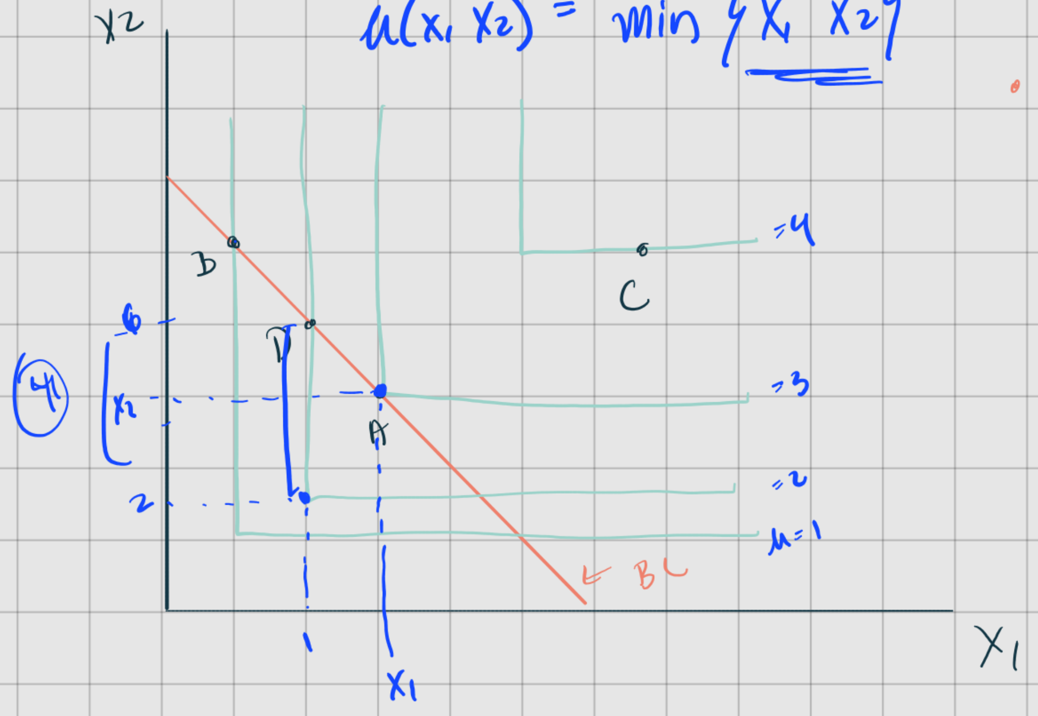
$\beta = 1$ $\quad P_2 = 2$

$$\frac{5}{1} \qquad \frac{2}{2} \qquad \left(\frac{m}{2}, 0\right)$$

$$[5 > 2]$$

Optimal choice for Perfect compliments (Kink sol^n)

$u(x_1, x_2) = \min\{x_1 \ x_2\}$



• Why are b+d not optimal?

Steps:

1) identify the utility / problem
2) optimal condition (kink) ⇒ min points are equal eq #1
3) write B.C Eq #2
4) Solve for x_1 + x_2 (2 equations + 2 unknowns)

$u(x_1, x_2) = \min\{\alpha x_1, \beta x_2\}$ P_1 P_2 m

1) Define Problem

$$\max_{x_1, x_2} \min\{\alpha x_1, \beta x_2\} \text{ s.t } P_1 x_1 + P_2 x_2 = m$$

2) Compare Proportion

$\alpha x_1 = \beta x_2$

$x_1 = \frac{\beta x_2}{\alpha}$

$\min\{1, 100\}$

3) Plug into B.C + solve for x_2

$P_1\left(\frac{\beta x_2}{\alpha}\right) + P_2 x_2 = m$

$x_2\left(\frac{P_1 \beta}{\alpha} + P_2\right) = m$

$x_2 = \frac{m}{\frac{P_1\beta}{\alpha} + P_2\left(\frac{\alpha}{\alpha}\right)} = \frac{m}{\frac{P_1\beta + P_2\alpha}{\alpha}}$

$$\boxed{x_2^* = \frac{m\alpha}{P_1\beta + P_2\alpha}}$$

4) Solve for x_1^*

$x_1^* = \frac{\beta}{\cancel{\alpha}} \cdot \frac{m\cancel{\alpha}}{P_1\beta + P_2\alpha}$

$= \frac{m\beta}{P_1\beta + P_2\alpha}$

Specific point is where the utility kinks

$\Rightarrow \therefore$ optimal (x_1^*, x_2^*)

$$(x_1^*, x_2^*) = \left(\frac{m\beta}{P_1\beta + P_2\alpha}, \frac{m\alpha}{P_1\beta + P_2\alpha}\right)$$

HW #2 : DUE Friday @ 11:59 pm #2 4 6

#2) $u(x_1, x_2) = 4x_1 + 3x_2$ ← perfect sub

(a) optimal Bundle:

1) max $4x_1 + 3x_2$ s.t $p_1 x_1 + p_2 x_2 = m$

2) $MU_1 = 4$, $MU_2 = 3$ → $|MRS| = \boxed{\frac{4}{3}}$

3) MRS vs PR

$\frac{4}{3}$ vs p_1/p_2

$>$ ⇒ $m/p_1, 0$ — A

$<$ $0, m/p_2$ B

$=$ $x \in [0, m/p_1]$ C

(b) $P_1 = 2$, $P_2 = 3$, $m = 60$ ⇒ $\frac{4}{3}$ vs $\frac{2}{3}$

all a good 1

$>$ ∴ $x_1^* = \frac{60}{2} = 30$

$x_2^* = 0$

(30, 0)

(#3) $u(B, C) = \boxed{6\ln(B) + 3\ln(C)}$

monotonic trans of CD

1) what type of Preference ⇒ $\boxed{u = B^6 C^3}$ - Cobb douglas.

2) $MRS = -\frac{MU_B}{MU_C}$

$\frac{\partial u(B,C)}{\partial B}$

$MU_B = 6B^5C^3$

$MU_C = 3B^6C^2$

$\frac{\partial u(B,C)}{\partial C}$

$- \frac{6B^5C^3}{3B^6C^2} = \boxed{\frac{-6C}{3B}}$

3) Optimal Bundle

max $u = B^6C^3$ st $P_B B + P_C C = m$

$$B^* = \boxed{\frac{a}{b+a}\frac{m}{P_1}} = \frac{6}{6+3}\frac{m}{P_B} = \boxed{\frac{2}{3}\frac{m}{P_B}}$$

$$C^* = \boxed{\frac{b}{b+a}\frac{m}{P_2}} = \frac{3}{6+3}\frac{m}{P_C} = \boxed{\frac{1}{3}\frac{m}{P_C}}$$

prop

4) fraction of total expen on Berries + choco.

$\frac{2}{3}$ on B + $\frac{1}{3}$ on C

proparsion consistent w coefficients of CD... characteristic of CD

5) $m = 300$

$P_B = 20$

$P_C = 10$

$$B = \frac{2}{3}\frac{300}{20} = 10$$

$$C = \frac{1}{3}\frac{300}{10} = 10$$

#6) $\boxed{u = \min(2x_1, 2x_2)}$

1) optimal bundle

1) max $u = \min(2x_1, 2, x_2)$ st $P_1 X_1 + P_2 X_2 = m$

2) $2x_1 = 2X_2$

$X_1 = \frac{2x_2}{2} = \boxed{X_1 = X_2}$

3) $P_1 X_1 + P_2 (X_1) = m$

$P_1 + P_2 (X_1) = m$

$$\boxed{X_1^* = \frac{m}{P_1 + P_2}} = X_2^*$$

2) proportion of consumption: $x_1 = x_2$

$$\frac{x_1^*}{x_2^*} = \frac{m/p_1+p_2}{m/p_1+p_2} = 1$$ (equal proportions)

↳ determined when setting equal

$\left(\frac{1}{2}x_1, 1x_2\right)$

3) $u = \min\left(x_1/2, x_2\right)$

$$\max \min\left(x_1/2, x_2\right) \quad st \quad p_1x_1 + p_2x_2 = m$$

$10x_1 \Rightarrow 5u$

$10x_2 = 10$

$\frac{20}{2} = 10$

$$\boxed{\frac{x_1}{2} = x_2}$$

$$p_1x_1 + p_2\left(\frac{x_1}{2}\right) = m$$

$$p_1 + \frac{p_2}{2}(x_1) = m$$

$$\boxed{x_1^* = \frac{2m}{2p_1+p_2}}$$

$$x_2^* = \frac{2m}{2p_1+p_2} \cdot \frac{1}{2}$$

$$\boxed{= \frac{m}{2p_1+p_2}}$$

Proportion

$$\frac{x_1^*}{x_2^*} = \frac{\frac{2m}{2p_1+p_2}}{\frac{m}{2p_1+p_2}} = 2$$

2 times as many x_1 to have same u as x_2